

2020 TEST 3

MATHEMATICS METHODS Year 12

Section One: Calculator-free

Your name		<u>:</u>
Teacher's name		·

Time and marks available for this section

Reading time before commencing work:

2 minutes

Working time for this section:

15 minutes

Marks available:

15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

- 1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet using a blue/black pen. Do not use erasable or gel pens.
- 3. Answer all questions.
- 4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- 5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 7. It is recommended that you do not use pencil, except in diagrams.

3

Question 1

(3 marks)

Consider the following discrete probability distribution for the random variable X.

x	1	2	3	4	5
P(X = x)	р	2p	3p	4p	5p

(a) Find the value of p.

(1 mark)

(b) Hence, find E(X).

(6 marks)

(a) If $\frac{dy}{dx} = e^{3x}$, then find an expression for, y, if y = 1 when x = 0.

(2 marks)

(b) Determine $\frac{d}{dx}(5x^2e^{x^3})$. There is no need to simplify your answer.

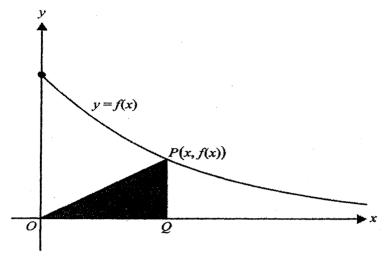
5

(c)

Determine $\int_0^{\frac{\pi}{6}} 2\cos(3x) dx$

(6 marks)

Let $f(x) = 2e^{-\frac{x}{5}}$. A right-angled triangle OQP has point O at the origin, point Q on the x - axis and pont P on the graph of f, as shown. The coordinates of P are (x, f(x)).



(a) Find an expression for the area, A, of the triangle OQP in terms of x. (2 marks)

(b) Find the maximum area of triangle OQP and the value of x for which the maximum occurs. (4 marks)

MATHEMATICS	METHODS Year	12
MULTIPMUTICS	MILITIODS I CAI	

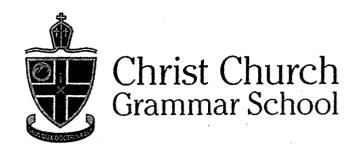
CALCULATOR-FREE

Additional working space

Question number:

Additional working space

Question number: _____



2020 TEST 3

MATHEMATICS METHODS Year 12

Section Two:
Calculator-assumed

Your name	
•	
Teacher's name	

Time and marks available for this section

Reading time before commencing work:

3 minutes

Working time for this section:

30 minutes

Marks available:

30 marks

Materials required/recommended for this section To be provided by the supervisor

This Question/Answer Booklet Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items:

drawing instruments, templates, and up to three calculators approved

for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

- 1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet using a blue/black pen. Do not use erasable or gel pens.
- 3. Answer all questions.
- 4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- 5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 7. It is recommended that **you do not use pencil**, except in diagrams.

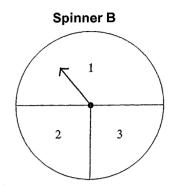
Question 4 (5 marks)

Two different spinners are constructed by dividing a circular card into 3 sectors scoring 1, 2 and 3. Each spinner has a rotating pointer pivoted at the centre, as shown below.

(Diagrams are not drawn to scale)

Spinner A

1
2
3



Spinner A has angles 135° , 90° and 135° for the sectors scoring 1, 2 and 3 respectively. **Spinner B** has angles 180° , 90° and 90° for the sectors scoring 1, 2 and 3 respectively. After being set in motion, the pointers come to rest independently in random positions. The random variable X is the **larger** of the two scores if they are different, and their common value if they are the same.

(a) Find P(X = 2).

(3 marks)

(b) Complete the probability distribution for X.

x	1	2	3
P(X = x)			

(6 marks)

The table shows the probability distribution of a discrete random variable \boldsymbol{X} .

x	1	2	3	4
P(X = x)	$\frac{p}{2}$	2 <i>p</i>	p + 0.1	p

Calculate:

(a) the value of p.

(1 mark)

(b) $E(X^2)$

(1 mark)

(c) E(2X + 1)

(2 marks)

(d) Var(2X + 1)

(7 marks)

Scientists are studying a complex biological process. They have measured the **rate of change** of concentration of two enzymes, A and B, over a period of time and have found that these rates are approximately modelled by the two functions

Enzyme A:
$$R_A'(t) = 4e^{-0.5t}, t \ge 0$$

Enzyme *B*:
$$R_B'(t) = \frac{8}{t^2+2}, t \ge 0$$

Where $R_A{}'(t)$ is the rate of change of concentration of enzyme A and $R_B{}'(t)$ is the rate of change of concentration of enzyme B after t minutes. Both rates of change of concentrations are measured in grams per litre per minute.

(a) Find the initial rate of change of concentration of each of the enzymes. (2 marks)

(b) State the derivative of $R_A'(t)$

(1 mark)

(c) Determine if the functions $R_A'(t)$ and $R_B'(t)$ are increasing, decreasing or neither for t > 0. (2 marks)

(d) State the time(s), to 4 decimal places, where $R_A{}'(t) = R_B{}'(t)$ for 0 < t < 10. (2 marks)

(5 marks)

A new casino gambling game is being developed. It costs \$5 to play a game. When two dice are rolled, if the uppermost numbers total 10 or more, or 4 or less, you are paid \$10. (Your \$5 cost to play and a further \$5) If the total score of 7 occurs, you receive your \$5 back. All other outcomes result in the loss of your \$5. If X represents the amount won,

(a)	complete t	the probability	distribution	for X
· /				

(3 marks)

x		
P(X = x)		

(b) calculate the expected amount you will win or lose if you play 6 games.

7

Question 8 (3 marks)

At the corner cafe, 70% of customers order a cappuccino and 30% order a latte. Of the customers who order a cappuccino, 60% order scrolls, and 25% of customers who order a latte order the scrolls.

Determine the probability that, if a scroll is ordered, the customer also ordered a cappuccino. Write your answer as a simplified fraction.

Question 9 (2 marks)

A discrete random variable X has the probability function $P(X = x) = k(1 - k)^x$ where x > 0. Write an expression for P(X > 1) in terms of k, leaving your answer in simplified form.

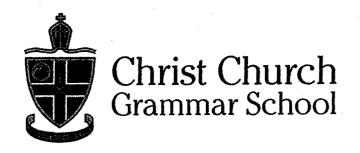
If
$$y = e^{\cos(x)}$$
 and $\frac{dx}{dt} = \frac{-1}{\cos(x)}$, then find the value of $\frac{dy}{dt}$ when $x = \frac{\pi}{3}$.

Additional working space

Question number: _____

Additional working space

Question number: _____



2020 TEST 3

MATHEMATICS METHODS Year 12

Section One: Calculator-free

Your name	· SOLUTIONS	•	
Teacher's name			

Time and marks available for this section

Reading time before commencing work:

2 minutes

Working time for this section:

15 minutes

Marks available:

15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items:

nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

- 1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet using a blue/black pen. Do not use erasable or gel pens.
- 3. Answer all questions.
- 4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- 5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 7. It is recommended that **you do not use pencil**, except in diagrams.

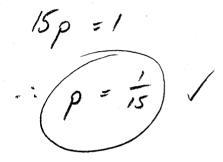
(3 marks)

Consider the following discrete probability distribution for the random variable X.

x	1	2	3	4	5
P(X = x)	р	2p	3p	4p	5p

(a) Find the value of p.

(1 mark)



(b) Hence, find E(X).

(2 marks)

[or 2 mks for just answ.]

(6 marks)

(a) If
$$\frac{dy}{dx} = e^{3x}$$
, then find an expression for, y , if $y = 1$ when $x = 0$. (2 marks)

$$\int e^{3x} dx = \frac{e^{3x}}{3} + C \qquad (Anti-diff)$$

$$1 = \frac{e^{3}}{3} + C$$

$$C = \frac{2}{3}$$

$$y = \frac{1}{3}e^{3x} + \frac{2}{3} \qquad (Answ)$$

(b) Determine $\frac{d}{dx}(5x^2e^{x^3})$. There is no need to simplify your answer.

$$\frac{dy}{dx} = \frac{10x \cdot e^{x^{3}} + 5x^{2} \cdot (3x^{2})e^{x^{3}}}{\sqrt{(v'v)}}$$
or
$$5x e^{x^{3}} (2+3x^{3})$$
(not required)

Question 2 continued

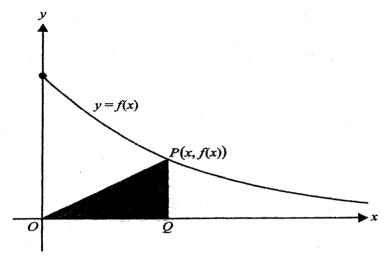
Determine $\int_0^{\frac{\pi}{6}} 2\cos(3x) dx$ (c)

$$= \frac{2\sin(3x)}{3} \sqrt{(Anli-Diff)}$$

$$= \frac{2\sin\frac{\pi}{2}}{3} - 0$$

(6 marks)

Let $f(x) = 2e^{-\frac{x}{5}}$. A right-angled triangle OQP has point O at the origin, point Q on the x - axis and pont P on the graph of f, as shown. The coordinates of P are (x, f(x)).



(a) Find an expression for the area, A, of the triangle OQP in terms of x. (2 marks)

$$A = \frac{1}{2}b \times h$$

$$A = \frac{1}{2}x \times 2e^{-x/5}$$

$$A = x \cdot e^{-x/5}$$

$$A = x \cdot e^{-x/5}$$

(b) Find the maximum area of triangle OQP and the value of x for which the maximum occurs. $v' \lor + \lor v'$ (4 marks)

$$\frac{dA}{dx} = \frac{e^{-\frac{1}{3}}}{e^{-\frac{1}{3}}} + x(-\frac{1}{5}e^{-\frac{1}{3}})$$

$$= e^{-\frac{1}{3}}(1 - \frac{1}{3})$$

$$\frac{dA}{dx} = 0 \Rightarrow x = 5$$

$$\frac{dA}{dx} = 0 \Rightarrow x = 5$$

$$(x - value)$$
When $x = 5 \Rightarrow A = 5e^{-1}$
End of questions
$$\sqrt{(correct \ value)}$$

M/	ATHEN	ΛA	TICS	METHO	DS	Year	12
----	-------	----	------	--------------	----	------	----

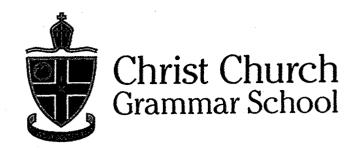
7

CALCULATOR-FREE

Additional working space

Question number:

Additional working space	Α	١dd	itic	onal	working	space
--------------------------	---	-----	------	------	---------	-------



2020 TEST 3

MATHEMATICS METHODS Year 12

Section Two: Calculator-assumed

Your name	· DOLUTIONS .		
Teacher's name			

Time and marks available for this section

Reading time before commencing work:

3 minutes

Working time for this section:

30 minutes

Marks available:

30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,

correction fluid/tape, eraser, ruler, highlighters

Special items:

drawing instruments, templates, and up to three calculators approved

for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

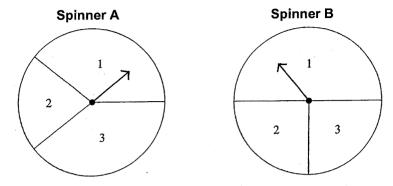
- 1. The rules of conduct of the CCGS assessments are detailed in the Reporting and Assessment Policy. Sitting this assessment implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet using a blue/black pen. Do not use erasable or gel pens.
- 3. Answer all questions.
- 4. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- 5. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
- 7. It is recommended that **you do not use pencil**, except in diagrams.

(2 marks)

Question 4 (5 marks)

Two different spinners are constructed by dividing a circular card into 3 sectors scoring 1, 2 and 3. Each spinner has a rotating pointer pivoted at the centre, as shown below.

(Diagrams are not drawn to scale)



Spinner A has angles 135° , 90° and 135° for the sectors scoring 1, 2 and 3 respectively. **Spinner B** has angles 180° , 90° and 90° for the sectors scoring 1, 2 and 3 respectively. After being set in motion, the pointers come to rest independently in random positions. The random variable X is the **larger** of the two scores if they are different, and their common value if they are the same.

(b) Complete the probability distribution for X.

 $P(X = x) = \frac{3}{\sqrt{8}} \times \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{32}} = \frac{3}{\sqrt{32}} = \frac{17}{\sqrt{32}} = \frac{3}{\sqrt{15}} = \frac{3}$

(6 marks)

The table shows the probability distribution of a discrete random variable X.

x	1	2	3	4
P(X = x)	$\frac{p}{2}$	2 <i>p</i>	p + 0.1	p

Calculate:

(a) the value of p.

$$\frac{p}{2} + 2p + p + 0.1 + p = 1 \Rightarrow (p = 0.2)$$

(b)
$$E(X^2)$$

 $E(x^2) = 1^2 + 2^2 + 2^2 \times 2p + 3^2 \times (p+o,1) + 4^2 p$

(1 mark)

(2 marks)

(c) E(2X + 1)

$$\mathcal{E}(x) = 2.6$$

$$CPad \cdot \sqrt{E(x)}$$

$$E(2x+1) = 2(2.6) + 1$$

$$= 6.2 \qquad (E(2x+1))$$

(d) Var(2X + 1) (2 marks)

$$0 = 0.9165151$$
 CPad.
 0.84 (VAR)
 0.84 0.84 0.84 0.84 0.84 0.84 0.84

See next page

(7 marks)

Scientists are studying a complex biological process. They have measured the rate of change of concentration of two enzymes, A and B, over a period of time and have found that these rates are approximately modelled by the two functions

Enzyme *A*:
$$R_A'(t) = 4e^{-0.5t}, t \ge 0$$

Enzyme *B*:
$$R_{B}'(t) = \frac{8}{t^2+2}, t \ge 0$$

Where $R_A{}'(t)$ is the rate of change of concentration of enzyme A and $R_B{}'(t)$ is the rate of change of concentration of enzyme B after t minutes. Both rates of change of concentrations are measured in grams per litre per minute.

Find the initial rate of change of concentration of each of the enzymes. (2 marks) (a)

$$R_{A}(0) = \underbrace{4}_{g/L/min} \mathcal{I}$$

$$R_{B}(0) = \underbrace{4}_{g/L/min} \mathcal{I}$$

State the derivative of $R_A'(t)$ (b)

(1 mark)

$$R_{A}^{"}(t) = \frac{-2e^{-t/2}}{\sqrt{2}}$$

Determine if the functions $R_A{}^\prime(t)$ and $R_B{}^\prime(t)$ are increasing, decreasing or neither (c) (2 marks) for t > 0.

$$R_{A}(t) \Rightarrow Decreasing /$$
 $R_{B}(t) \Rightarrow Decreasing /$

State the time(s), to 4 decimal places, where $R_A{}'(t) = R_B{}'(t)$ for 0 < t < 10. (d)

(2 marks)

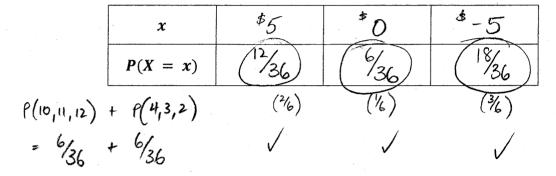
See next page

(5 marks)

A new casino gambling game is being developed. It costs \$5 to play a game. When two dice are rolled, if the uppermost numbers total 10 or more, or 4 or less, you are paid \$10. (Your \$5 cost to play and a further \$5) If the total score of 7 occurs, you receive your \$5 back. All other outcomes result in the loss of your \$5. If *X* represents the amount won.

(a) complete the probability distribution for X.

(3 marks)



(b) calculate the expected amount you will win or lose if you play 6 games.

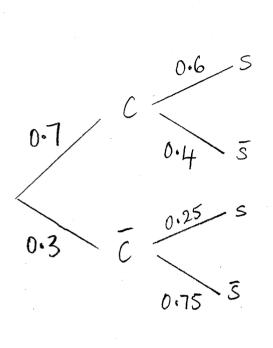
$$E(x) = 5(\frac{2}{6}) + o(\frac{1}{6}) - 5(\frac{3}{6})$$

$$= -\frac{5}{6} \qquad \sqrt{E(x)}$$
ie losing \$\frac{3}{6}\$ per game
$$\therefore $\frac{4}{5}$ loss for 6 games $\sqrt{Answ}$$$

(3 marks)

At the corner cafe, 70% of customers order a cappuccino and 30% order a latte. Of the customers who order a cappuccino, 60% order scrolls, and 25% of customers who order a latte order the scrolls.

Determine the probability that, if a scroll is ordered, the customer also ordered a cappuccino. Write your answer as a simplified fraction.



$$P(c|s) = \frac{P(cns)}{P(s)}$$

$$\sqrt{(num)} = \frac{0.7 \times 0.6}{0.7 \times 0.6 + 0.3 \times 0.25}$$

$$\frac{0.42}{0.495}$$

$$= \frac{28}{33} / (simplified)$$

$$fraction$$

Question 9

(2 marks)

A discrete random variable X has the probability function $P(X = x) = k(1 - k)^x$ where x > 0. Write an expression for P(X > 1) in terms of k, leaving your answer in simplified form.

e an expression for
$$P(X > 1)$$
 in terms of k , leaving your answer in simplified

$$P(X > 1) = \left| - \left(P(X = 1) \right) \right| \quad \text{Given } X > 0$$

$$= \left| - \left(K(I - K)' \right) \right| \quad \text{(complement in } k's)$$

$$= \left| - \left(K - K^2 \right) \right|$$

$$= \left| K^2 - K + 1 \right| \quad \text{(simplified)}$$

$$\sigma^R = K(K - 1) + 1$$
See next page

(2 marks)

If $y = e^{\cos(x)}$ and $\frac{dx}{dt} = \frac{-1}{\cos(x)}$, then find the value of $\frac{dy}{dt}$ when $x = \frac{\pi}{3}$.

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{but}{dx} = -\sin(x) e^{\cos(5x)}$$

$$\frac{dy}{dt} = -\sin(x) e^{\cos(x)} \times \left(\frac{1}{\cos(x)}\right)$$

$$\frac{dy}{dt} = \frac{\sin(x)}{\cos(x)} e^{\cos(x)}$$

$$\frac{dy}{dt} = \frac{\sin(x) e^{\cos(x)}}{\cos(x)}$$

$$\frac{dy}{dt} = \frac{\tan(x) e^{\cos(x)}}{\cos(x)}$$

2